

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-12487

U.S.S. Maginsky
(2)

ST- LPS- RWR-10619

REFLECTION FACTOR FROM THE PLANE-STRATIFIED MEDIUM
OF LUNNITE FOR VARIOUS ANGLES OF INCIDENCE OF
ELECTROMAGNETIC WAVES

by

Yu. G. Matveyev

(USSR)

FACILITY FORM 602

67 30041	
(ACCESSION NUMBER)	(THRU)
9	0
(PAGES)	(CODE)
CR-85265	30
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

6 JUNE 1967

REFLECTION FACTOR FROM THE PLANE-STRATIFIED MEDIUM
OF LUNNITE FOR VARIOUS ANGLES OF INCIDENCE OF
ELECTROMAGNETIC WAVES

(*)

Astronomicheskiy Zhurnal
Tom 44, No.2, 419-425,
Izdatel'stvo "NAUKA", 1967

by Yu. G. Matveyev

ABSTRACT

The reflection factor from a nonuniform inhomogeneous medium of plane-stratified nature is computed for the case when this medium's index of refraction varies from a certain value on the surface to another value in depth along the exponential curve.

It is shown that the angular dependence of the reflection factor of such a medium coincides well with that of an homogeneous medium for a certain effective value of the index of refraction, determined from the condition of equality of these media's reflections factors when the incidence of electromagnetic waves is normal. Inasmuch as the reflection factor of a stratified medium is dependent on wavelength, the effective reflection factor is also a function of λ .

An approximation is obtained of the frequency dependence of the effective index of refraction as a function of inhomogeneity depth. The applicability is concluded of Fresnel formulas for the calculation of integral Moon's radioemission regardless of the existence of well known inhomogeneity of properties in depth.

*
* *

As is well known, the integral radioemission of the Moon is observed in most of cases. This is why for the interpretation of experimental data the elaboration of a corresponding theory is required; it has been worked out in the works under ref. [1-4]. What must be known in particular is the distribution law of the reflection factor along the lunar disk, which amounts to the requirement of knowledge of reflection factor's dependence on the angle of incidence of the waves.

(*) Koeffitsient OTRAZHENIYA OT POSKOSLOISTOY SREDY LUNITA DLYA RAZLICH-
NYKH UGLOV PADENIYA ELEKTROMAGNITNYKH VOLN.

In connection with the inhomogeneity of properties in depth revealed at the surface of the Moon [5, 6], there arises the question of calculation of reflection factor's dependence upon the wave's incidence angle for an inhomogeneous plane-stratified medium, and of ascertaining to what extent it may depart from the earlier utilized dependence for a uniform medium described by the Fresnel formula. This is precisely the object of the present work. X

The case was considered in [6] of normal wave incidence in one-dimensional inhomogeneous nonabsorbing medium. The accounting of absorption for the lunar surface does not affect the result, for the wave damping in an inhomogeneous medium is found to be negligibly small. The calculation of the reflection factor from such a medium is performed in the present work as a function of the angle of incidence applicably to two construction models of Moon's upper layer considered in ref. [6].

In this calculation we shall utilize the equation for reflection factors [7], written for its easy solution on a computer in the form

$$\begin{aligned}\frac{dR}{dx} &= \Gamma(x)(1 - R^2) \cos \varphi, \\ \frac{d\varphi}{dx} &= -2\beta(x) - \frac{\Gamma(x)}{R}(1 + R^2) \sin \varphi,\end{aligned}\quad (1)$$

where R is the modulus of the reflection factor, ϕ is its phase, $\beta(x) = k_0[n^2(x) - \sin^2 i]$, $\Gamma(x) = q'/2q$, $q = \beta/m$ ($m = 1$ for the horizontally polarized wave, $m = n^2(x)$ for a vertically polarized wave) $q' = dq/dx$, i is the incidence angle of the wave, $k(0)$ is the wave number. The index of refraction $n(x)$ is written in the form [6]

$$n(x) = 1 + n_0[1 - (1 - \xi)e^{-x/x_0}]. \quad (2)$$

According to the considerations stated in the work [6], the initial condition for the solution of Eqs(1) is written in the form

$$\begin{aligned}R_{\text{нач}} &= \int_{\infty}^{x_0} \Gamma(x) dx, \\ \varphi_{\text{нач}} &= \pi.\end{aligned}\quad (3)$$

the subscript 'нач' standing for 'initial'

We compiled in Table 1 the results of calculations of the reflection factor by the power of R^2 for various angles of incidence i and wavelengths equal to 3, 10, 30 and 70 cm (for two models of Moon's upper layer construction [6]) for $x_0 = 1.5$, and in Table 2 for $x_0 = 3$ cm. The families of computed curves $R^2(i)$ for $x_0 = 1.5$ cm are plotted in Fig.1.

It is apparently appropriate to introduce an effective index of refraction that would allow us to determine the dependence $R^2(i)$ for various angles λ and x_0 computed by the Fresnel formulas

../..

$$R_{\perp}^2 = \left[\frac{\cos i - \sqrt{n_{\text{eff}}^2(\lambda, x_0) - \sin^2 i}}{\cos i + \sqrt{n_{\text{eff}}^2(\lambda, x_0) - \sin^2 i}} \right]^2,$$

$$R_{\parallel}^2 = \left[\frac{n_{\text{eff}}^2(\lambda, x_0) \cos i - \sqrt{n_{\text{eff}}^2(\lambda, x_0) - \sin^2 i}}{n_{\text{eff}}^2(\lambda, x_0) \cos i + \sqrt{n_{\text{eff}}^2(\lambda, x_0) - \sin^2 i}} \right]^2, \quad (4)$$

without resorting to the solution of the system of differential equations (1). R_{\perp}^2 and R_{\parallel}^2 are respectively the reflection factors for the horizontally and vertically polarized wave.

The effective value of the index of refraction is found from the condition of equality of the reflection factors computed by (4) and (1) for a normal wave incidence; then

$$n_{\text{eff}}^2(\lambda, x_0) = \left[\frac{1 + R^2}{1 - R^2} + \sqrt{\left(\frac{1 + R^2}{1 - R^2} \right)^2 - 1} \right]^2. \quad (5)$$

where R^2 is the computed value of the reflection factor from (1), which must be taken from the table compiled in the work [6]. The results of calculations of $n_{\text{eff}}^2(\lambda, x_0)$ are compiled in Table 3. The reflection factor in the function of i and $n_{\text{eff}}^2(\lambda, x_0)$ for $\lambda = 10, 30, 70$ cm and $x_0 = 1.5$ cm, computed by formulas (4) is plotted by dots in Figs. 1 and 2, where the exact values of $R(i)$ computed by (1) are also brought out. Comparison of the results of calculations of the dependence of the reflection factor on the angle of incidence of the wave shows that for a stratified medium, this dependence is described with sufficient precision by the Fresnel formulas for the equivalent value of the index of refraction determined from (5). The greatest discrepancy constitutes 5 to 4 percent. Utilizing the dependence of R^2 on λ , obtained by calculation from (1), we may write an empirical formula for the effective value of the index of refraction in the form

$$n_{\text{eff}}^2(\lambda, x_0) = n_3^2 - (n_3^2 - n_2^2) e^{-a\lambda/x_0}, \quad (6)$$

where n_2 and n_3 are respectively the indices of refraction on the surface and in depth of the layer, a is a factor found from the condition

$$e^{-a\lambda_{\text{av}}/x_0} = 0.5;$$

λ_{av} being equal to 30–35 cm, is the average wavelength of the transitional interval in which the reflection factor varies from its minimum value in centimeter waves to the maximum value at the borderline of decimeter waves (see [6]). Consequently,

$$a = \frac{-x_0 \ln 0.5}{\lambda_{\text{av}}} = \frac{0.7 x_0}{\lambda_{\text{av}}} \approx 0.033. \quad (7)$$

TABLE 5

λ, cm	$\epsilon_2 = 1.7; \epsilon_3 = 2.25$						$\epsilon_2 = 2.25; \epsilon_3 = 3.2$					
	$x_0 = 1 \text{ cm}$		$x_0 = 1.5 \text{ cm}$		$x_0 = 2 \text{ cm}$		$x_0 = 1.5 \text{ cm}$		$x_0 = 2 \text{ cm}$		$x_0 = 3 \text{ cm}$	
	(*)	(**)	(*)	(**)	(*)	(**)	(*)	(**)	(*)	(**)	(*)	(**)
1	1.74	1.72	1.74	1.71	1.74	1.71	2.25	2.28	2.25	2.27	2.25	2.26
2	1.78	1.73	1.78	1.72	1.78	1.71	2.25	2.31	2.25	2.28	2.25	2.27
3	1.78	1.75	1.78	1.73	1.78	1.72	2.27	2.34	2.27	2.29	2.27	2.28
4	1.80	1.77	1.78	1.73	1.78	1.72	2.30	2.37	2.27	2.31	2.27	2.29
6	1.84	1.80	1.80	1.75	1.78	1.74	2.31	2.42	2.27	2.34	2.27	2.31
10	1.91	1.85	1.84	1.78	1.78	1.76	2.41	2.52	2.30	2.39	2.27	2.35
20	2.05	1.97	1.97	1.90	1.84	1.81	2.65	2.71	2.53	2.50	2.36	2.41
30	2.13	2.05	2.05	1.95	1.91	1.85	2.81	2.85	2.66	2.63	2.43	2.52
40	2.17	2.10	2.10	1.99	1.97	1.90	2.90	2.95	2.79	2.69	2.55	2.59
60	2.20	2.17	2.17	2.13	2.05	1.97	2.97	3.07	2.81	2.82	2.66	2.71
100	2.22	2.23	2.20	2.13	2.15	2.07	3.03	3.16	2.81	3.00	2.83	2.89
200	2.25	2.25	2.25	2.23	2.22	2.19	3.05	3.20	3.05	3.16	3.00	3.00

$\epsilon_{0\phi\phi}$ stands for ϵ_{eff} (experim) (see (*), odd columns)

$\epsilon_{0\phi\phi, \text{BMT}}$ stands for ϵ_{eff} (computed) (see (**), even columns)

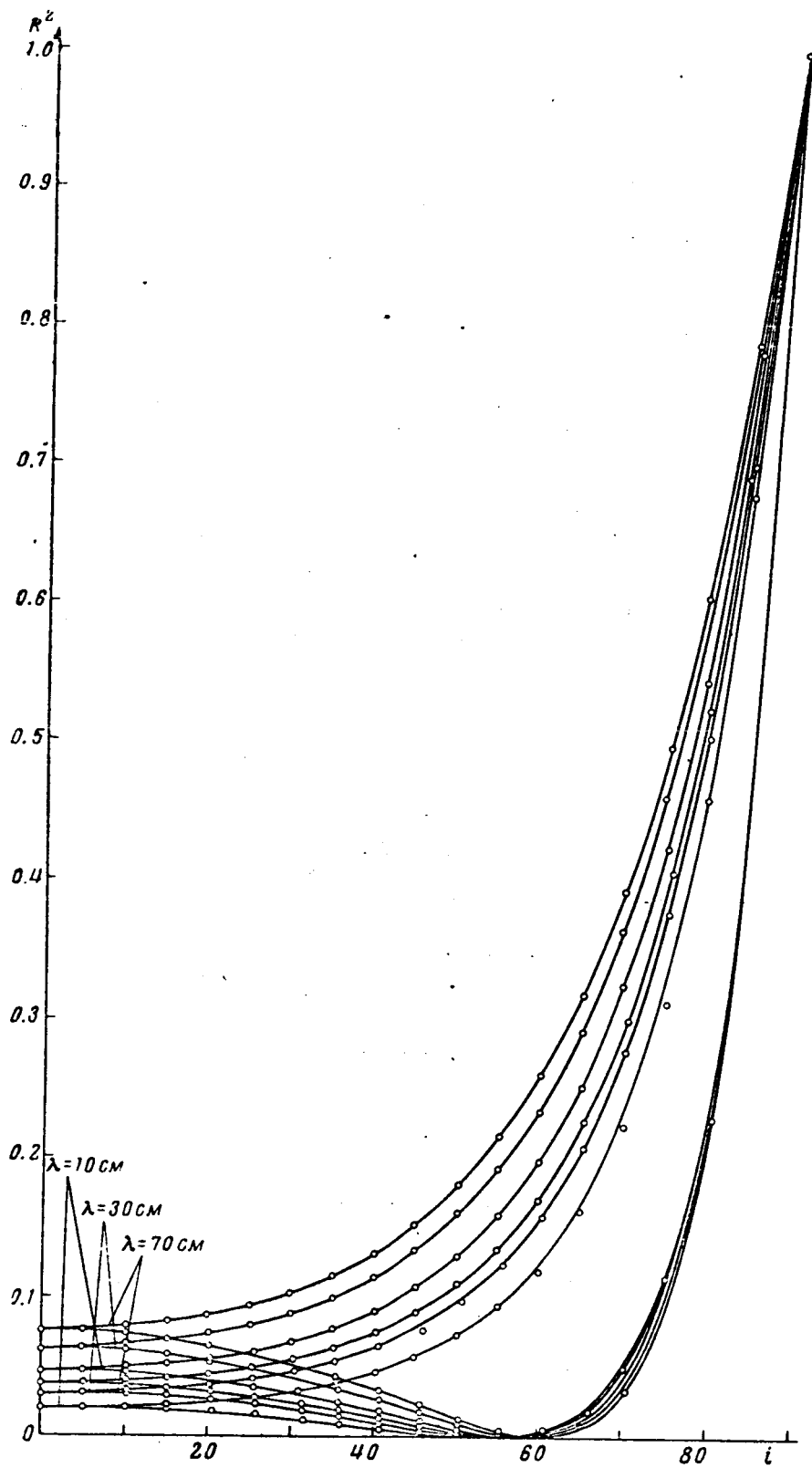


Fig.1

The values of $n_{300}^2(\lambda, x_0)$ computed by formula (6), agree well with the values obtained from (5) (see Table 3).

Therefore, expressions (4) and (6) allow us to compute the reflection factor $R^2(i)$ for various values of λ , x_0 , n_2^2 and n_3^2 of a plane-parallel nonabsorbing medium at least in the range of investigated values of x_0 and n .

In conclusion I wish to express my gratitude to V. S. Troitskiy, R. E. Erm and I. I. Malova for their help in the work.

***** T H E E N D *****

REFERENCES

1. V. S. TROITSKIY. Astronom. Zhurnal., 31, 6, 1954.
2. V. D. KROTIKOV, V. S. TROITSKIY. Ibid. 39, 6, 1962.
3. V. D. KROTIKIV. Radiofizika, 6, 5, 1963.
4. V. D. KROTIKOV. Ibid. 3, 8, 1965.
5. V. S. TROITSKIY. Astronom. Zh., 41, 4, 1964.
6. A. M. BREKHOVSKIY. Volny v sloistyykh sredakh (Waves in Stratified Media). Izd-vo A.N. SSSR, 1957.

Institute of Radiophysics
of the
Gor'kiy State University

Manuscript received on
1 April 1966

Contract No. NAS-5-12487
VOLT TECHNICAL CORPORATION
1145 19th St. NW
WASHINGTON D.C. 20036.
Tel: 225-6700; 225-4930.

Translated by ANDRE L. BRICHANT
on 5 June 1967

DISTRIBUTION

GSFC 611 (6); 612 (3); 614 (6); 615 (6); 613 (8); 640 (3); 641 (6), 620, 622
rest as usual

SG (usual); SL BRYSON, MOLLOY, WILMARTH, BRUNK, FELLOWS, HIPSHER, HOROWITZ
SM FOSTER, GILL; SA BADGLEY, rest as usual

A R C, La R C (see ST-535), M S C, TA HESS (3), TH (INGELS). RICE UNIV. Br. O'BRIEN
J.P.L. T-122 (3), 186-133 (3), VIS. LAB. WYCKOFF, 111-113 BOOTHE. 125-109A LAWSON.
plus the usual permanent addressees.